

A copula-based approach for modelling temporal dependence in financial data

Un approccio basato sulle copule per modellare la dipendenza in una serie finanziaria

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Riassunto: In questo lavoro proponiamo di modellare la dipendenza in una serie di rendimenti finanziari utilizzando una funzione copula BB7 con due parametri di associazione e marginali Log-Dagum. La metodologia suggerita è applicata ad alcune serie finanziarie.

Keywords: Archimedean copula, financial returns, Markov process

1. Introduction

Several *stylized facts* which characterize financial markets have been the focus of recent studies. Many of these studies pointed out that the empirical distribution of financial returns is far from the Normal model since it tends to be asymmetric and with heavy tails. Moreover, for financial time series it is rarely realistic to assume independence between consecutive observations. For example, financial data may show a temporal dependence structure of its volatilities which is often referred to as *volatility clustering*: large (small) absolute returns tend to follow large (small) absolute returns. Indeed, different forms of nonlinear dependence may exist which need to be properly modelled.

Motivated by these considerations, we propose to model the dependence between two consecutive returns Y_t and Y_{t-1} using, in a Markov chain framework, a copula approach with marginals that are skewed and with heavy tails. In particular, we use a two-parameters Archimedean copula, called BB7, since one-parameter copulas cannot distinguish among different types of tail dependence. In our case, one BB7 parameter (say θ) allows us to capture upper tail dependence whereas the other (say δ) the lower tail dependence. Moreover, we model the marginal returns through the Log-Dagum distribution, obtained by a logarithmic transformation of the well-known Dagum random variable, whose *cdf* is $F(x; \beta, \lambda, \nu) = (1 + \lambda e^{-\nu x})^{-\beta}$, $x \in \mathbb{R}$ and $\beta, \lambda, \nu > 0$. Log-Dagum distribution shows heavy tails and it may be both symmetric and asymmetric (positive or negative).

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2. The copula approach

Let the series of returns $\{Y_t : t = 1, 2, \dots, T\}$ be a first order stationary Markov chain with continuous state space. The transition *cdf* based on copula approach is defined as:

$$H(y_t/y_{t-1}; \alpha, \theta, \delta) = C_{2/1}(F(y_t; \alpha)/F(y_{t-1}; \alpha); \theta, \delta)$$

where $C_{2/1}(v/u; \theta, \delta)$ is the first derivative of the copula $C(u, v; \theta, \delta)$ with respect to u , with $u = F(y_t; \alpha)$, $v = F(y_{t-1}; \alpha)$ (Joe, 1997; Nelsen, 1999); besides, α and θ, δ are, respectively, the marginal and association parameters. We assume that $F(y_t; \alpha)$ is the Log-Dagum *cdf* with $\alpha = (\beta, \lambda, \nu)$ and the copula is the Archimedean BB7 family, defined as $C(u, v; \theta, \delta) = \varphi^{-1}(\varphi(u) + \varphi(v))$ with generator function $\varphi(t) = [1 - (1 - t)^\theta]^{-\delta} - 1$, $\theta > 1$, $\delta > 0$. The copula approach allows us to deal with some real characteristics of the time series of returns separately (Chen and Fan, 2006). Indeed, we choose the copula approach to take into account the temporal dependence regardless of the marginal distributions which, instead, are chosen to describe the marginal behavior of Y_t and Y_{t-1} allowing for heavy tails and skewness.

The model we propose is applied to real data. For our analysis we consider the daily returns of four Italian stocks: *Generali* from 8/01/1999 to 7/01/2004; *Banca Popolare Italiana* (Bpi) from 2/01/1995 to 7/08/2006; *Telecom* from 8/01/1999 to 7/01/2004; *Tiscali* from 27/10/1999 to 19/05/2005.

The log-likelihood is $\ell(\theta, \delta; \alpha) = \sum_{t=1}^T \log f(y_t; \alpha) + \sum_{t=2}^T \log c(F(y_t; \alpha), F(y_{t-1}; \alpha); \theta, \delta)$

where $f(\cdot; \cdot)$ is the Log-Dagum density function and $c(\cdot, \cdot; \cdot, \cdot)$ is the copula density defined as the second derivative of $C(u, v; \theta, \delta)$ with respect to u and v . Since the likelihood function cannot be maximized analytically, we have implemented a numerical procedure for estimating the parameters of the model. The maximum likelihood (ML) estimates are shown in the following table, together with the estimated asymptotic variances. We observe that all the estimates are significative.

Table 1: ML estimates and asymptotic estimated variances (*in brackets*)

	β	λ	ν	θ	δ
Generali	0.9553 (0.008)	1.0105 (0.021)	100.22 (17.681)	1.0426 (0.0004)	0.0602 (0.001)
Telecom	0.9115 (0.007)	1.1084 (0.028)	69.101 (8.848)	1.0349 (0.001)	0.0395 (0.001)
Tiscali	1.2658 (0.014)	0.6222 (0.006)	50.48 (3.617)	1.0793 (0.0004)	0.1164 (0.001)
Bpi	1.000 (0.001)	0.9478 (0.0003)	128.90 (6.211)	1.0478 (0.0001)	0.0822 (0.0003)

References

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